Symbolic methods applied to the automation of computational proofs

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November 20,2017

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Introduction to security

Security ?

Why is computer security important ?

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Stuxnet

Security ?

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Stuxnet - Took control of Iranian nuclear power plants

Security ?

Boom !

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Ok, but why is the security of everyday services important ?
Mails, Facebook, Twitter, ...

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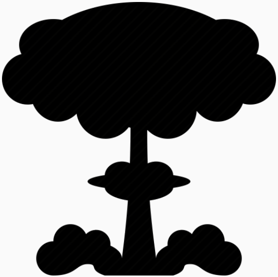

Boom !

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- E-mails
- Internet search history
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$\hookrightarrow$ Should I care if a company or a government can read my mails ?

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What an attacker could learn about you:

- Do you cheat your spouse ?
- Are you homosexual ? Your son ?
- Are you in need of money ? Are you sick ?

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It matters!

- Blackmail and corruption
- Commercial targeting
- Harassment and segregation
- Freedom of speech

Conclusion of the Introduction

We want security !

Conclusion of the Introduction

We want formal proofs of security !

Symbolic model

Proofs by saturation

1. Define exactly which operations the attacker can perform.
2. Define the security of our protocol/scheme.
3. Try all possible attacker actions, until we:
    - either break security,
    - or get a security proof.

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Realm

- Messages are abstract terms: $enc(message, sk)$
- Equationnal theory captures the attacker power:

$$dec(enc(m, sk), sk)) = m$$

- The attacker can intercept everything sent over the network

A symbolic method example

Deducibility
Given a set of messages, can an attacker deduce a secret ?

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Example

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Computational model

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1. Assume that some problem is computationally difficult
2. Define the security of our protocol/scheme
3. Show that if one can break the security, one can break the difficult problem

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Realm

- Messages are bitstrings
- Attackers are any PPT algorithm/TM

Computational vs Symbolic

Fight !

Symbolic model

- Network controlled by the attacker
- Primitives are idealized

Computational model

- Network controlled by the attacker
- Arbitrary PPT attacker

Fight !

Symbolic model

- Network controlled by the attacker
- Primitives are idealized

✓ Many automated proofs

× No proofs by hand

× Missed attacks

Computational model

- Network controlled by the attacker
- Arbitrary PPT attacker

× Few automated proofs

× Hand made proofs hard to check

✓ Stronger proofs

$\hookrightarrow$ Our focus : use technics from symbolic models to improve automation in the computational model

A formal framework for computational proofs

Game equivalence

Goal example:

$$\forall \mathcal{A}.\ a, b : \mathbb{F}_q . \mathcal{A}(g^a, g^b, g^{ab}) \simeq a, b, c : \mathbb{F}_q . \mathcal{A}(g^a, g^b, g^c)$$

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Arbitraty PPT TM

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Randomly sampled over $\mathbb{F}_q$

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Equality of distribution

$\hookrightarrow$ The attacker cannot distinguish between the two inputs

Reduction example

The DDH assumption

$$\forall \mathcal{A}.\ (a, b : \mathbb{F}_q . \mathcal{A}(g^a, g^b, g^{ab})) \simeq (a, b, c : \mathbb{F}_q . \mathcal{A}(g^a, g^b, g^c))$$

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The simulator
We can replace any $\mathcal{A}$ by:

$$B(\mathcal{A})(e_1, e_2, e_3) := d : \mathbb{F}_q, \mathcal{A}(e_1, e_2, g^d, e_3^d)$$

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$\hookrightarrow$ We want to do this in reverse, i.e build the B

Automated construction of simulators

The problem

Question
Given an assumption and a goal, can we automatically find B?

Example

Simulator validity

Assumption: $\forall \mathcal{A}.\ (a, b : \mathbb{F}_q.\mathcal{A}(g^a, g^b, g^{ab})) \simeq (a, b, c : \mathbb{F}_q.\mathcal{A}(g^a, g^b, g^c))$

Goal: $a, b, c : \mathbb{F}_q.\mathcal{A}(g^a, g^b, g^c, g^{abc}) \simeq a, b, c, d : \mathbb{F}_q.\mathcal{A}(g^a, g^b, g^c, g^{dc})$

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Potential simulator B

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Given (g^a, g^b, g^{ab}), is it possible to compute (g^a, g^b, g^c, g^{abc}) ?

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$\hookrightarrow$A deducibility problem

Main idea

Left hand side of an assumption :

$$x_1, ..., x_n : \mathbb{F}_q.\mathcal{A}(e_1, .., e_k)$$

Left hand side of a goal:

$$x_1, ..., x_n, x_{n+1} ..., x_{n+k} : \mathbb{F}_q.\mathcal{A}(t_1, \ldots, t_l)$$

Check, if for all terms t_i, $1 \leq i \leq l$:

$$e_1, ..., e_k \vdash t_i$$

Correctness of using deducibility

Disadvantage
Something not deducible in the symbolic word might be deducible in the computational world.

$$enc(a, sk), enc(b, sk) \nvdash enc(a + b, sk)$$

Advantage
If something is deducible in the symbolic world, it is always deducible.

$\hookrightarrow$ We may find valid simulators using deducibility.

Option 1
Use existing results in the symbolic model for simple theories.

Using deducibility in practice

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$\hookrightarrow$ provides fast automation.

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Extend the symbolic technics to more faithful theories.

Using deducibility in practice

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Option 2
Extend the symbolic technics to more faithful theories.

$\hookrightarrow$ provides slower but complete automation.

Contributions

Existing work

- Deducibility only for polynomials of degree one in the exponent, without axioms (e,g $a \neq 0$)
- AutoGnP [Barthe et al, CCS15] used heuristics to construct simulators

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Contributions

- Axioms ($a \neq 0$)
- Bilinear maps
- Any polynomials in the exponent
- Matrices

Symbolic Proofs for Lattice-Based Cryptography, CCS18

G. Barthe, X. Fan, J. Gancher, B. Gregoire, C. Jacomme, E. Shi

Conclusions

Conclusion regarding the presented work

A complete procedure

Given an assumption and a goal, we provide a complete procedure to decide if the assumption can be applied.

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WIP
Use symbolic methods (deducibility, static equivalence, unification):

- to automatize more complex crypto proofs (RND rule)
- to verify masking schemes
- to handle multistage games, oracle games, ...

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- to automatize more complex crypto proofs (RND rule)
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Conclusion regarding my PhD

Multiple projects in parallel

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Try to get the best of both worlds:

- Use symbolic methods to enhance automation in the computational world.

  G. Barthe, B. Gregoire, S. Kremer, P-Y.Strub

- Composing proofs of security in the computational world[1].

  H. Comon-Lundh, G. Scerri

- Case studies[2] in the symbolic world, as exhaustive as possible.

  S. Kremer

[1] prove the security of big protocols by only proving its components.

[2] multi-factor authentication protocols, SSH