Symbolic methods in computational cryptography proofs

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Introduction

Secure protocols

Security proofs

- Precise definitions of security;
- precise modelling of the protocol;
- clear assumptions.

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A classical example: RSA-OAEP
From 1994 to 2010, one proof, 5 different papers.

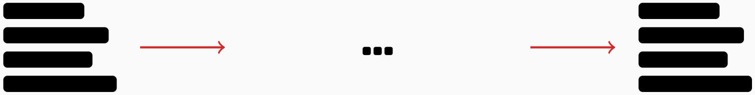
A new hope

V. Shoup, 2004

Security proofs in cryptography may be organized as sequences of games [...] this can be a useful tool in taming the complexity of security proofs that might otherwise become so messy, complicated, and subtle as to be nearly impossible to verify

A new hope

Game sequences



A new hope

Realistic model of the protocol.

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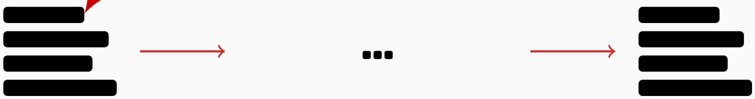

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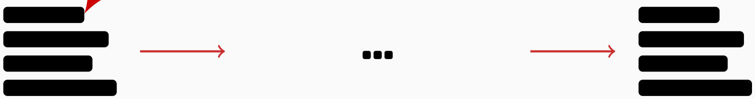

Idealized protocol, where security is clear.

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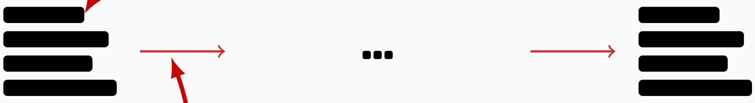

Small, verifiable, transformation.

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Proofs should be easily verifiable, because only based on small transformations.

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Game sequences
Proofs are still long and difficult to verify entirely for concrete schemes.

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- but this kind of proof is suited for computer-aided verification.

A new new hope

Mechanized provers
CryptoHol, CryptoVerif, Easycrypt, FCF . . .

A new new hope

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An interactive prover to write formal proofs through game sequences.

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The current challenge

Intuition **VS** **EasyCrypt**

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Our general goal

Automation
Reduce distance between pen and paper proofs and Easycrypt proofs.

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Game transformations
Three important ingredients:

- Uniformity
- Independence
- Equivalence of distribution

Uniformity
Does a message follow the uniform distribution ?

$\hookrightarrow$ the attacker learns nothing

Computational proofs

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Does a message depend on the distribution of some secret ?

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Equivalence
Do two messages have the same probability distribution ?

$\hookrightarrow$ same attacker behaviour

Goal

Precise goal

Decide uniformity, independence and equivalence for simple programs.

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Simple programs ?

- **inputs/outputs**
- **datatypes** (booleans/bitstrings, $\mathbb{F}_q$, DH exponentiation)
- **constructs** (random sampling, conditionals, bindings)

An example

Easycrypt snippet: Cramer Shoup Key generation

$$x \xleftarrow{\$} \mathbb{F}_q \setminus \{0\}$$
$$y, z \xleftarrow{\$} \mathbb{F}_q$$
$$gx, gy, gz \leftarrow g^x, g^y, g^z$$
$$x_1, x_2, y_1, y_2, z_1, z_2 \xleftarrow{\$} \mathbb{F}_q$$
$$g_1, a, a_1 \leftarrow gx, gy, gz$$
$$k \xleftarrow{\$} dk$$
$$e \leftarrow g^{x1} * g_1^{x2}$$
$$f \leftarrow g^{y1} * g_1^{y2}$$
$$h \leftarrow g^{z1} * g_1^{z2}$$
$$\text{return } pk \leftarrow (k, g, g\_, e, f, g)$$
$$\text{return } sk \leftarrow (k, g, g\_, x_1, x_2, y_1, y_2, z_1, z_2)$$

Easycrypt snippet: Cramer Shoup Key generation

Uniform sampling in a finite field.

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Variable assignment.

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The EasyCrypt goal

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Is pk independent from x_2, y_2 and z_2 ?

The probabilistic question

Does this expression follow the uniform distribution?

$$(k,\ x,\ x_1 + x * x_2,\ x_2,\ y_1 + x * y_2,\ y_2,\ z_1 + x * z_2,\ z_2)$$

An intuitive characterization

Bijections

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$$f(u, v, w) \simeq (u, v, w) \Leftrightarrow f \text{ is a bijection}$$

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Is this function a bijection?

$$(k, x, x_1, x_2, y_1, y_2, z_1, z_2) \mapsto$$
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- $x_1 + x * x_2 \; - x * x_2 = x_1$

Symbolic methods

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$\hookrightarrow$ Use symbolic methods to perform proofs in the computational model.

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Deducibility

- Can an attacker deduce a secret ?
- Always correct (a symbolic attack is a computational attack)
- Not always computationally complete (may miss attacks).

$\hookrightarrow$ We only need the correction to have a witness of uniformity.

A general Framework

Programs

Variables

- A set $X = (x, y, z, \dots)$ of deterministic variables;
- a set $R = (u, v, w, \dots)$ of random variables.

Programs
A program is a sequence of terms built over $t \in \mathcal{T}(\Sigma, X \uplus R)$.

Programs examples

Examples

- $P(\{x, y\}, \{u\}) = (x + u, y, xy)$
- $P(\{x, y\}, \{u, v, w\}) = (uv + vw + wu + xy)$

Programs examples

Input : x,y

Sample uniformly u

Return $(x+u, y, xy)$

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Probabilistic relations

The framework

Terms and Programs: $P_1(X, R) \in \mathcal{T}(\Sigma, X \uplus R)$

$$P(X, R) = P_1(X, R), \ldots, P_k(X, R)$$

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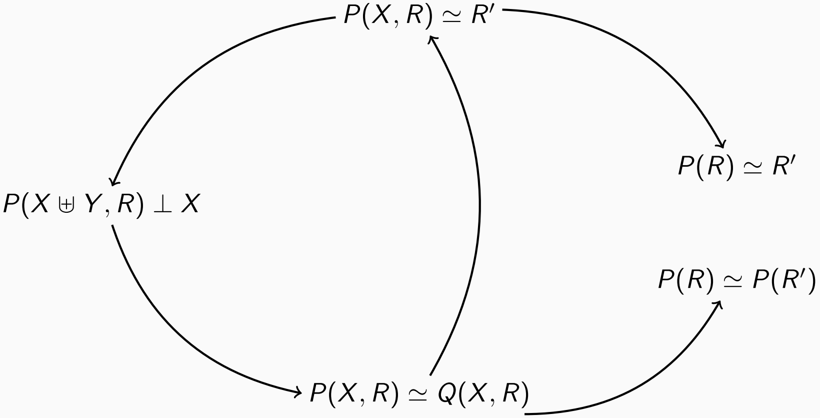
$$P(X,R) = P_1(X,R), \ldots, P_k(X,R)$$

Relations

Uniformity	$P(X,R) \simeq R$
Independence	$P(X,R) \perp R$
Equivalence	$P(X,R) \simeq Q(X,R)$

Encodings


$P(X, R) \simeq R'$
$P(R) \simeq R'$
$P(X \uplus Y, R) \perp X$
$P(R) \simeq P(R')$
$P(X, R) \simeq Q(X, R)$

Summary of the symbolic abstractions

Deduction
Uniformity for $P(X, R)$ of length $|R| \Leftrightarrow$ Deduction.

Unification and deduction constraints
Equivalence $\Leftrightarrow$ unification and deduction constraints (with private homomorphic symbol).

Static equivalence
Equivalence $\Rightarrow$ static equivalence.

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Equivalence $\Rightarrow$ static equivalence.

$\hookrightarrow$ We obtain connections with widely studied questions

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Easy to derive heuristics

We can use over and under approximations of the equational theories.

- If a program follows the uniform distribution when sampling over a ring of characteristic two, it also does when sampling over any $\mathbb{F}_{2^q}$.
- If two programs are not equivalent when sampling over $\mathbb{F}_2$, they are not equivalent over a ring of characteristic two.

The abstraction

Modular
There are many combination results for symbolic methods.

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There are many combination results for symbolic methods.

- Easy to add support for free function symbols, or bilinear pairings, or any disjoint equational theories.

Implementation

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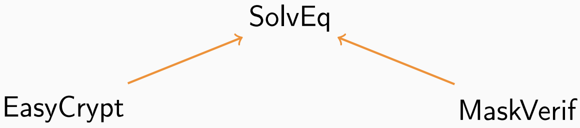

- handles deduction and static equivalence in rings and finite fields;
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Sample of Cramer Shoup proofs

```
swap{1} 16 -9;  wp; swap -1; swap -1.
rnd (fun z ⇒  z + G1.w{2} * G1.z2{2})
(fun z ⇒  z - G1.w{2} * G1.z2{2}).
rnd.
wp; swap -1.
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rnd; wp; rnd; wp.
rnd (fun z ⇒  z / x{1}) (fun z ⇒  z * x{1}) ⇒  /=.
```

Sample of Cramer Shoup proofs

17 tactic calls replaced by a single tactic, with content extracted from cryptographic intuition.

```
rndmatch
    (z1, G1.z, fun z => z + G1.w{2} * G1.z2{2})
    (z2, G1.z2)
    (y1, G1.y, fun z => z + G1.w{2} * G1.y2{2})
    (y2, G1.y2)
    (x1, G1.x, fun z => z + G1.w{2} * G1.x2{2})
    (x2, G1.x2)
    (k , G1.k )
    (z , x , fun z => z / x{1})
    (y , G1.u )
    (x , G1.w ).
```

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Improvements

- Witnesses of negative results
- New examples not covered by the old heuristic

Conclusion

Final words

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- Link different probabilistic problems;
- abstracted into term algebras;
- derive algorithms from symbolic methods that are principled, sound and/or complete;
- implement and integrate the resulting algorithms inside existing tools.

Final words

Future work

- automate the application of cryptographic assumptions;
- automate the verification of MPC protocols;
- find an efficient algorithm for general equivalence in finite fields.