Automates d'arbre

TD n°3 : Relations

October 10th, 2019

Exercise 1 : Closure properties - back to basics

1. Given a recognizable relation, show that all its cylindrifications and projections are recognizable. (provides explicit trees automatas)
2. Is the domain and the image of a binary relation recognizable ?
3. Given R,R' binary relations, show that $R \circ R'$ is recognizable.
4. Give an example of a n-ary relation such that its ith projection followed by its ith cylindrification does not give back the original relation.
5. On the contrary, show that ith cylindrification followed by ith projection gives back the original relation.

Exercise 2 : Some relations

1. Let $\mathcal{F} = \{0(2), 1(2), n(0)\}$. Give an automaton recognizing :
   $R_1 = \{(t, t')|t, t' \in T(\mathcal{F}), Pos(t) = Pos(t') \ \wedge \forall p \in Pos(t), t(p) = 1 \Rightarrow t'(p) = 1\}$
2. Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Is the relation $R_2 = \{(g(t), t) \mid t \in T(\mathcal{F})\}$ recognizable ? And if $\mathcal{F} = \{g(1), a(0)\}$?
3. Here assume that $\mathcal{F} = \{g(1), a(0)\}$. Is R_2^* recognizable ?
4. Is $R_3 = \{(t, f(t, t')) \mid t, t' \in T(\mathcal{F})\}$ recognizable ?
5. Design two relations, one recognizable and one which is not. Challenge your classmates with them.

Exercise 3 : Rewriting systems

Let $\mathcal{F} = \{a_i(1) \mid 1 \leq i \leq n\} \cup \{0(0)\}$.

1. Prove that any rewrite system $\rightarrow$ (i.e. the one step rewriting relation) on $\mathcal{F}$ is recognizable.
2. Prove that $S = \{(a_1^k(a_1(a_2(a_2^l(0)))), a_1^k(a_2^l(0))) \mid k, p \in \mathbb{N}\}$ is recognizable.
3. Prove that S^* is not recognizable.